

Расчетно-графическая работа 4.2

Функции комплексной переменной

1. Вычислить $a + b$, ab , a/b , если:

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|-----------------------------------|-----------------------------------|-----------------------------------|
| 1. $a = 3 + 2i$, $b = 2 - 2i$. | 2. $a = 3 - 2i$, $b = 2 + 2i$. | 3. $a = 3 + 2i$, $b = 2 + 2i$. |
| 4. $a = 3 - 2i$, $b = 2 - 2i$. | 5. $a = 2 + 3i$, $b = 2 - i$. | 6. $a = 2 + 3i$, $b = 2 + i$. |
| 7. $a = 2 - 3i$, $b = 2 - i$. | 8. $a = 2 - 3i$, $b = 2 + i$. | 9. $a = 2 + i$, $b = 3 - 2i$. |
| 10. $a = 2 + i$, $b = 3 + 2i$. | 11. $a = 2 - i$, $b = 3 - 2i$. | 12. $a = 2 - i$, $b = 3 + 2i$. |
| 13. $a = 4 + 3i$, $b = 2 - 3i$. | 14. $a = 4 + 3i$, $b = 2 + 3i$. | 15. $a = 4 - 3i$, $b = 2 - 3i$. |
| 16. $a = 4 - 3i$, $b = 2 + 3i$. | 17. $a = 1 + 2i$, $b = 2 + 3i$. | 18. $a = 1 + 2i$, $b = 2 - 3i$. |
| 19. $a = 1 - 2i$, $b = 2 + 3i$. | 20. $a = 1 - 2i$, $b = 2 - 3i$. | 21. $a = 3 + 3i$, $b = 2 - i$. |
| 22. $a = 3 + 3i$, $b = 2 + i$. | 23. $a = 3 - 3i$, $b = 2 - i$. | 24. $a = 3 - 3i$, $b = 2 + i$. |
| 25. $a = 2 + 2i$, $b = 3 - i$. | 26. $a = 2 + 2i$, $b = 3 + i$. | 27. $a = 2 - 2i$, $b = 3 + i$. |
| 28. $a = 2 - 2i$, $b = 3 - i$. | 29. $a = 2 + i$, $b = 3 + i$. | 30. $a = 2 + i$, $b = 3 - i$. |

2. Найти все значения корня.

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|--------------------------------|--|--|--------------------------------|---|
| 1. $\sqrt{-1}$. | 2. $\sqrt{\frac{-1 + i\sqrt{3}}{2}}$. | 3. $\sqrt{\frac{-1 - i\sqrt{3}}{2}}$. | 4. $\sqrt[4]{1}$. | 5. $\sqrt[4]{\frac{-1 + i\sqrt{3}}{32}}$. |
| 6. $\sqrt[3]{1}$. | 7. $\sqrt[3]{-1}$. | 8. $\sqrt[3]{-i}$. | 9. $\sqrt{-16}$. | 10. $\sqrt[3]{i}$. |
| 11. $\sqrt[3]{8}$. | 12. $\sqrt[3]{8i}$. | 13. $\sqrt[4]{16}$. | 14. $\sqrt{-8}$. | 15. $\sqrt[4]{\frac{-1 - i\sqrt{3}}{32}}$. |
| 16. $\sqrt{-8i}$. | 17. $\sqrt[4]{-\frac{1}{16}}$. | 18. $\sqrt[4]{-8 + i8\sqrt{3}}$. | 19. $\sqrt[3]{\frac{1}{8}}$. | 20. $\sqrt[3]{\frac{i}{8}}$. |
| 21. $\sqrt[4]{\frac{1}{16}}$. | 22. $\sqrt[4]{-8 - i8\sqrt{3}}$. | 23. $\sqrt[3]{\frac{1}{8}}$. | 24. $\sqrt[3]{-\frac{i}{8}}$. | 25. $\sqrt[4]{-256}$. |
| 26. $\sqrt[3]{27}$. | 27. $\sqrt[4]{\frac{1}{256}}$. | 28. $\sqrt[4]{-\frac{1}{256}}$. | 29. $\sqrt[3]{\frac{i}{27}}$. | 30. $\sqrt[4]{256}$. |

3. Вычислить.

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|--|-------------------------------|------------------------|--------------------------------|-------------------------------------|
| 1. $\sin(2 + i)$. | 2. e^{1-i} . | 3. $\arcsin 2$. | 4. $\arccos 2$. | 5. $\arccos(-\frac{5}{4})$. |
| 6. $(1 - i)^{3-3i}$. | 7. $\arcsin \frac{\pi}{3}i$. | 8. $(-1)^{\sqrt{2}}$. | 9. $\ln(-i)$. | 10. $\arccos \frac{5}{4}$. |
| 11. $\arccos 3$. | 12. $\arcsin 3$. | 13. $\arcsin i$. | 14. $\arccos i$. | 15. $\ln(-3)$. |
| 16. $\operatorname{arctg} \frac{i}{3}$. | 17. 1^i . | 18. i^i . | 19. $\ln(4 + 3i)$. | 20. $\operatorname{arctg}(1 + i)$. |
| 21. $(\frac{1+i}{\sqrt{2}})^{2i}$. | 22. $\ln(1 + i)$. | 23. $i^{1/i}$. | 24. $\cos(1 + i)$. | 25. $\operatorname{sh}(1 + i)$. |
| 26. $\operatorname{ch}(1 + i)$. | 27. 2^{1+2i} . | 28. 2^i . | 29. $e^{\frac{\pi}{2}i} + 1$. | 30. $e^{1 - \frac{\pi}{2}i}$. |

4. Дана аналитическая функция $f(z) = u + iv$, где $u = \operatorname{Re}f(z)$, $v = \operatorname{Im}f(z)$.

Найти эту функцию, если:

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|-------------------------------|------------------------------|-------------------------------|
| 1. $u = x^2 + x - y^2$. | 2. $u = x^3 - x - 3xy^2$. | 3. $u = x^3 - 3xy^2 + 2x$. |
| 4. $u = x^2 - 2x - y^2$. | 5. $u = 2x^3 + x - 6xy^2$. | 6. $u = 2x^2 - x - 2y^2$. |
| 7. $u = 2x^3 - 3x - 6xy^2$. | 8. $u = 3x^3 + x - 9xy^2$. | 9. $u = 3x^2 + 2x - 3y^2$. |
| 10. $u = 3x^3 - 2x - 9xy^2$. | 11. $u = x^3 - 4x - 3xy^2$. | 12. $u = x^2 + 4x - y^2$. |
| 13. $u = 3x^3 - 3x - 9xy^2$. | 14. $u = 3x^2 + 3x - 3y^2$. | 15. $u = 4x^3 - x - 12xy^2$. |
| 16. $v = 2xy + y$. | 17. $v = 3x^2y - y - y^3$. | 18. $v = 3x^2y + 2y - y^3$. |
| 19. $v = 2xy - 2y$. | 20. $v = 6x^2y + y - 2y^3$. | 21. $v = 4xy - y$. |
| 22. $v = 6x^2y - 3y - 2y^3$. | 23. $v = 9x^2y + y - 3y^3$. | 24. $v = 6xy + 2y$. |
| 25. $v = 9x^2y - 2y - 3y^3$. | 26. $v = 3x^2y - 4y - y^3$. | 27. $v = 2xy + 4y$. |
| 28. $v = 9x^2y - 3y - 3y^3$. | 29. $v = 6xy + 3y$. | 30. $v = 12x^2y - y - 4y^3$. |

5. Найти образ E области D плоскости z при отображении функцией

$w = f(z)$.

1. $D: \{z \mid -\pi < \operatorname{Im} z < 0\}; w = e^z$.
2. $D: \{z \mid -2\pi < \operatorname{Im} z < -\pi\}; w = e^z$.
3. $D: \{z \mid 0 < \operatorname{Im} z < \pi\}; w = e^z$.
4. $D: \{z \mid \pi < \operatorname{Im} z < 2\pi\}; w = e^z$.
5. $D: \{z \mid \operatorname{Re} z < 0, 0 < \operatorname{Im} z < \pi\}; w = e^z$.
6. $D: \{z \mid \operatorname{Re} z < 0, \pi < \operatorname{Im} z < 2\pi\}; w = e^z$.
7. $D: \{z \mid \operatorname{Re} z > 0, 0 < \operatorname{Im} z < \pi\}; w = e^z$.
8. $D: \{z \mid \operatorname{Re} z > 0, 0 < \operatorname{Im} z < 2\pi\}; w = e^z$.
9. $D: \{z \mid \operatorname{Re} z < 0, 0 < \operatorname{Im} z < 2\pi\}; w = e^z$.
10. $D: \{z \mid -\infty < \operatorname{Re} z < +\infty, 0 < \operatorname{Im} z < \frac{\pi}{2}\}; w = e^z$.
11. $D: \{z \mid -\infty < \operatorname{Re} z < +\infty, \frac{\pi}{2} < \operatorname{Im} z < \pi\}; w = e^z$.
12. $D: \{z \mid \operatorname{Re} z > 0, 0 < \operatorname{Im} z < \frac{\pi}{2}\}; w = e^z$.
13. $D: \{z \mid \operatorname{Re} z > 0, \frac{\pi}{2} < \operatorname{Im} z < \pi\}; w = e^z$.
14. $D: \{z \mid \operatorname{Im} z > 0\}; w = \ln z$.
15. $D: \{z \mid |z| < 1; 0 < \arg z < 2\pi\}; w = \ln z$.
16. $D: \{z \mid |z| < 1; 0 < \arg z < \pi\}; w = \ln z$.
17. $D: \{z \mid |z| < 1; 0 < \arg z < 2\pi\}; w = \ln z$.
18. $D: \{z \mid 0 < \operatorname{Re} z < 1, 0 < \operatorname{Im} z < \pi\}; w = e^z$.
19. $D: \{z \mid 1 < \operatorname{Re} z < 2, 0 < \operatorname{Im} z < \pi\}; w = e^z$.
20. $D: \{z \mid |z| < 1; \pi < \arg z < 2\pi\}; w = \ln z$.
21. $D: \{z \mid |z| < 1; 0 < \arg z < \frac{\pi}{4}\}; w = z^4$.
22. $D: \{z \mid |z| < 3; 0 < \arg z < \frac{\pi}{4}\}; w = z^4$.
23. $D: \{z \mid |z| < 2; 0 < \arg z < \frac{\pi}{3}\}; w = z^3$.
24. $D: \{z \mid |z| > 2; 0 < \arg z < \frac{\pi}{3}\}; w = z^3$.
25. $D: \{z \mid |z| > 1; 0 < \arg z < \frac{\pi}{4}\}; w = z^4$.
26. $D: \{z \mid |z| > 3; 0 < \arg z < \frac{\pi}{3}\}; w = z^3$.

27. $D: \{z: \operatorname{Re} z > 0, 0 < \operatorname{Im} z < \pi\}; w = e^z.$
28. $D: \{z: 0 < \operatorname{Re} z < \pi, \operatorname{Im} z > 0\}; w = e^{iz}.$
29. $D: \{z: \operatorname{Re} z > 0, 0 < \operatorname{Im} z < \frac{\pi}{2}\}; w = e^{2z}.$
30. $D: \{z: \operatorname{Re} z < 0, 0 < \operatorname{Im} z < \frac{\pi}{3}\}; w = e^{3z}.$

6. Найти функцию, отображающую область D плоскости z на область E плоскости w .

1. $D: \{z: \operatorname{Re} z < 1, 0 < \operatorname{Im} z < 2\}; E: \{w: \operatorname{Im} w > 0\}.$
2. $D: \{z: -1 < \operatorname{Re} z < 0, \operatorname{Im} z > 0\}; E: \{w: \operatorname{Im} w > 0\}.$
3. $D: \{z: -1 < \operatorname{Re} z < 0, \operatorname{Im} z < 0\}; E: \{w: \operatorname{Im} w > 0\}.$
4. $D: \{z: 0 < \operatorname{Re} z < \pi, \operatorname{Im} z > 0\}; E: \{w: \operatorname{Im} w > 0\}.$
5. $D: \{z: |z| < 1, \operatorname{Im} z > 0\}; E: \{w: \operatorname{Im} w > 0\}.$
6. $D: \{z: |z| < 2, 0 < \arg z < \frac{\pi}{4}\}; E: \{w: \operatorname{Im} w > 0\}.$
7. $D: \{z: |z| < \frac{1}{2}, 0 < \arg z < \frac{\pi}{3}\}; E: \{w: \operatorname{Im} w > 0\}.$
8. $D: \{z: |z| < 3, 0 < \arg z < \frac{\pi}{4}\}; E: \{w: \operatorname{Im} w > 0\}.$
9. $D: \{z: 0 < \operatorname{Re} z < 1\}; E: \{w: \operatorname{Im} w > 0\}.$
10. $D: \{z: 0 < \operatorname{Re} z < 4\}; E: \{w: \operatorname{Im} w > 0\}.$
11. $D: \{z: 0 < \operatorname{Re} z < \frac{1}{2}\}; E: \{w: \operatorname{Im} w > 0\}.$
12. $D: \{z: |z| < 1, 0 < \arg z < \frac{\pi}{4}\}; E: \{w: \operatorname{Im} w > 0\}.$
13. $D: \{z: |z| < 3, 0 < \arg z < \frac{\pi}{6}\}; E: \{w: 0 < \operatorname{Re} w < 1, -\infty < \operatorname{Im} w < +\infty\}.$
14. $D: \{z: |z| < 1, 0 < \arg z < \frac{\pi}{4}\}; E: \{w: 0 < \operatorname{Re} w < 1, -\infty < \operatorname{Im} w < +\infty\}.$
15. $D: \{z: |z| < 2, 0 < \arg z < \frac{\pi}{3}\}; E: \{w: 0 < \operatorname{Re} w < \pi, -\infty < \operatorname{Im} w < +\infty\}.$
16. $D: \{z: |z| < 2, 0 < \arg z < \frac{\pi}{2}\}; E: \{w: 0 < \operatorname{Re} w < 1, -\infty < \operatorname{Re} w < +\infty\}.$
17. $D: \{z: |z| < 2, |z-1| > 1\}; E: \{w: 0 < \operatorname{Im} w < 1, -\infty < \operatorname{Re} w < +\infty\}.$
18. $D: \{z: |z| < 2, |z+1| > 1\}; E: \{w: 0 < \operatorname{Im} w < \pi, -\infty < \operatorname{Re} z < +\infty\}.$
19. $D: \{z: |z| < 1, \operatorname{Re} z > 0\}; E: \{w: 0 < \operatorname{Im} w < \pi, -\infty < \operatorname{Re} w < +\infty\}.$
20. $D: \{z: |z-2| < 1, \operatorname{Im} z > 0\}; E: \{w: \operatorname{Im} z > 0\}.$
21. $D: \{z: |z-2i| < 1, \operatorname{Re} z < 0\}; E: \{w: \operatorname{Im} w > 0\}.$
22. $D: \{z: |z+2| < 1, \operatorname{Im} z > 0\}; E: \{w: \operatorname{Im} w > 0\}.$
23. $D: \{z: 0 < \operatorname{Re} z < 3, \operatorname{Im} w > 0\}; E: \{w: \operatorname{Im} w > 0\}.$
24. $D: \{z: -2 < \operatorname{Re} z < 0, \operatorname{Im} z < 0\}; E: \{w: \operatorname{Im} w > 0\}.$
25. $D: \{z: |z| < \frac{1}{2}, 0 < \arg z < \frac{\pi}{4}\}; E: \{w: \operatorname{Im} w > 0\}.$
26. $D: \{z: |z| < 3, 0 < \arg z < \frac{\pi}{3}\}; E: \{w: \operatorname{Im} w > 0\}.$
27. $D: \{z: |z+i| > 1, |z+2i| < 2\}; E: \{w: \operatorname{Im} w > 0\}.$
28. $D: \{z: \operatorname{Re} z > 0, 0 < \operatorname{Im} z < \frac{\pi}{3}\}; E: \{w: \operatorname{Im} w > 0\}.$
29. $D: \{z: |z| < \frac{1}{3}, 0 < \arg z < \frac{\pi}{3}\}; E: \{w: \operatorname{Im} w > 0\}.$
30. $D: \{z: |z-i| > 1, |z| < 2\}; E: \{w: 0 < \operatorname{Im} w < \pi, -\infty < \operatorname{Re} w < +\infty\}.$

7. Вычислить интеграл.

$$\textcircled{1} \int_{AB} z^2 dz; AB: \{y = x^2, z_A = 0, z_B = 1+i\}.$$

2. $\int_L (z+1)e^z dz; L: \{|z|=1, \operatorname{Re} z \geq 0\}$.
3. $\int_{AB} \operatorname{Im} z^3 dz; AB: \{\text{отрезок прямой}, z_A=0, z_B=2+2i\}$.
4. $\int_{AB} (z^2+7z+1) dz; AB: \{\text{отрезок прямой}, z_A=1, z_B=1-i\}$.
5. $\int_{ABC} |z| dz; ABC: \{\text{ломаная}, z_A=0, z_B=-1+i, z_C=1+i\}$.
6. $\int_{AB} \bar{z}^2 dz; AB: \{\text{отрезок прямой}, z_A=0, z_B=1+i\}$.
7. $\int_{AB} (12z^5+4z^3+1) dz; AB: \{\text{отрезок прямой}, z_A=1, z_B=i\}$.
8. $\int_{ABC} z^3 e^{z^4} dz; ABC: \{\text{ломаная}, z_A=i, z_B=1, z_C=0\}$.
9. $\int_{ABC} \operatorname{Re} \frac{\bar{z}}{z} dz; AB: \{|z|=1, \operatorname{Im} z \geq 0\}$,
 $BC: \{\text{отрезок прямой}, z_B=1, z_C=2\}$.
10. $\int_L \frac{\bar{z}}{z} dz; L: \text{граница области } \{1 < |z| < 2, \operatorname{Re} z > 0\}$.
11. $\int_{ABC} (z^2 + \cos z) dz; ABC: \{\text{ломаная}, z_A=0, z_B=1, z_C=i\}$.
12. $\int_{ABC} (\operatorname{ch} z + \cos iz) dz; ABC: \{\text{ломаная}, z_A=0, z_B=-1, z_C=i\}$.
13. $\int_L |z|\bar{z} dz; L: \{|z|=4, \operatorname{Re} z \geq 0\}$.
14. $\int_L (\operatorname{ch} z + z) dz; L: \{|z|=1, \operatorname{Im} z \leq 0\}$.
15. $\int_{AB} (3z^2+2z) dz; AB: \{y=x^2, z_A=0, z_B=1+i\}$.
16. $\int_L |z|\operatorname{Re} z^2 dz; L: \{|z|=R, \operatorname{Im} z \geq 0\}$.
17. $\int_L z\operatorname{Re} z^2 dz; L: \{|z|=R, \operatorname{Im} z \geq 0\}$.
18. $\int_{ABC} (z^2+1) dz; ABC: \{\text{ломаная}, z_A=0, z_B=-1+i, z_C=i\}$.
19. $\int_{AB} e^{|z|^2} \operatorname{Im} z dz; AB: \{\text{отрезок прямой}, z_A=1+i, z_B=0\}$.
20. $\int_L (\sin iz + z) dz; L: \{|z|=1, \operatorname{Re} z \geq 0\}$.

21. $\int_{AB} z \operatorname{Re} z^2 dz$; $AB : \{\text{отрезок прямой, } z_A = 0, z_B = 1 + 2i\}$.
22. $\int_{AB} (2z + 1) dz$; $AB : \{y = x^3, z_A = 0, z_B = 1 + i\}$.
23. $\int_{ABC} z \bar{z} dz$; $AB : \{|z| = 1, \operatorname{Re} z \geq 0, \operatorname{Im} z \geq 0\}$,
 $BC : \{\text{отрезок, } z_B = 1, z_C = 0\}$.
24. $\int_L (\cos iz + 3z^2) dz$; $L : \{|z| = 1, \operatorname{Im} z \geq 0\}$.
25. $\int_L |z| dz$; $L : \{|z| = \sqrt{2}, \frac{3\pi}{4} \leq \arg z \leq \frac{5\pi}{4}\}$.
26. $\int_{ABC} (z^9 + 1) dz$; $ABC : \{\text{ломаная, } z_A = 0, z_B = 1 + i, z_C = i\}$.
27. $\frac{1}{2i} \int_{z=R} \bar{z} dz$.
28. $\int_{ABC} (\sin z + z^5) dz$; $ABC : \{\text{ломаная, } z_A = 0, z_B = 1, z_C = 2i\}$.
29. $\int_{AB} z \operatorname{Im} z^2 dz$; $AB : \{\text{отрезок прямой, } z_A = 0, z_B = 1 + i\}$.
30. $\int_L (z^3 + \sin z) dz$; $L : \{|z| = 1, \operatorname{Re} z \geq 0\}$.

8. Вычислить при помощи формул Коши интеграл.

1. $\oint_{|z=2} \frac{\operatorname{ch} iz}{z^2 + 4z + 3} dz$.
2. $\oint_{|z-2|=3} \frac{e^{z^2}}{z^2 - 6z} dz$.
3. $\oint_{|z-2|=5} \frac{e^{z^2}}{z^2 - 6z} dz$.
4. $\oint_{|z-1|=\frac{1}{2}} \frac{e^{\frac{1}{z}}}{z^2 + z} dz$.
5. $\oint_{|z-2|=2} \frac{\operatorname{ch} z}{z^4 - 1} dz$.
6. $\oint_{|z|=1} \frac{e^z \cos \pi z}{z^2 + 2z} dz$.
7. $\oint_{|z-1-i|=1} \frac{\sin \pi(z-1)}{z^2 - 2z + 2} dz$.
8. $\oint_{|z|=1} \frac{\operatorname{tg} z}{ze^{\frac{1}{z^2}}} dz$.
9. $\oint_{|z|=3} \frac{\cos(z + \pi i)}{z(e^z + 2)} dz$.
10. $\oint_{|z|=5} \frac{dz}{z^2 + 16}$.
11. $\oint_{|z|=4} \frac{dz}{(z^2 + 9)(z + 9)}$.
12. $\oint_{|z|=2} \frac{\sin z \sin(z-1)}{z^2 - z} dz$.

13. $\oint_{|z|=2} \frac{\operatorname{ch} z}{(z+1)^3(z-1)} dz.$ 14. $\oint_{|z|=1} \frac{\cos z}{z^3} dz.$
15. $\oint_{|z|=1} \frac{\operatorname{sh} z^2}{z^3} dz.$ 16. $\oint_{|z-1|=1} \frac{\sin \frac{\pi}{4} z}{(z-1)^2(z-3)} dz.$
17. $\oint_{|z|=2} \frac{z \operatorname{sh} z}{(z^2-1)^2} dz.$ 18. $\oint_{|z-3|=6} \frac{z dz}{(z-2)^3(z+4)}$
19. $\oint_{|z-2|=3} \frac{\operatorname{ch} e^{inz}}{z^3-4z^2} dz.$ 20. $\oint_{|z|=\frac{1}{2}} \frac{1}{z^3} \cos \frac{\pi}{z+1} dz.$
21. $\oint_{|z-2|=1} \frac{e^{\frac{1}{z}}}{(z^2+4)^2} dz.$ 22. $\oint_{|z|=\frac{1}{2}} \frac{1-\sin z}{z^2} dz.$
23. $\oint_{|z-1|=\frac{1}{2}} \frac{e^{iz}}{(z^2-1)^2} dz.$ 24. $\oint_{|z-3i|=2} \frac{e^z}{z(z-2i)} dz.$
25. $\oint_{|z-i|=1} \frac{\cos z}{(z-i)^3} dz.$ 26. $\oint_{|z-1|=3} \frac{e^z}{z^2-z-2} dz.$
27. $\oint_{|z+2|=2} \frac{e^z}{z^2-z-2} dz.$ 28. $\oint_{|z-1|=2} \frac{e^z}{z^2-z-2} dz.$
29. $\oint_{|z|=1} \frac{e^z}{z(z-2i)} dz.$ 30. $\oint_{|z-i|=2} \frac{e^z}{z^2-z-2} dz.$

9. Найти все лорановские разложения по степеням z функции.

1. $w = \frac{z-2}{2z^3+z^2-z}$ 2. $w = \frac{z-4}{z^4+z^3-2z^2}$
3. $w = \frac{z-2}{2z^3+3z^2-9z}$ 4. $w = \frac{z-4}{z^4+2z^3-16}$
5. $w = \frac{z-2}{2z^3+5z^2-25z}$ 6. $w = \frac{z-4}{z^4+3z^3-18z^2}$
7. $w = \frac{z-2}{2z^3+7z^2-49z}$ 8. $w = \frac{z-4}{z^4+4z^3-32z^2}$
9. $w = \frac{z-2}{2z^3+9z^2-81z}$ 10. $w = \frac{z-4}{z^4+5z^3-50z^2}$
11. $w = \frac{z-2}{2z^3+11z^2-121z}$ 12. $w = \frac{z-4}{z^4+6z^3-72z^2}$
13. $w = \frac{z-2}{2z^3+13z^2-169z}$ 14. $w = \frac{z-4}{z^4+7z^3-98z^2}$
15. $w = \frac{z-2}{2z^3+15z^2-225z}$ 16. $w = \frac{z-4}{z^4+8z^3-128z^2}$
17. $w = \frac{z-2}{z+z^2-2z^3}$ 18. $w = \frac{z-4}{2z^2+z^3-z^4}$
19. $w = \frac{z-2}{9z+3z^2-2z^3}$ 20. $w = \frac{z-4}{8z^2+2z^3-z^4}$

$$\begin{array}{ll}
21. w = \frac{5z + 50}{25z + 5z^2 - 2z^3} & 22. w = \frac{3z + 36}{18z^2 + 3z^3 - z^4} \\
23. w = \frac{7z + 98}{49z + 7z^2 - 2z^3} & 24. w = \frac{32z^2 + 4z^3 - z^4}{5z + 100} \\
25. w = \frac{81z + 9z^2 - 2z^3}{11z + 242} & 26. w = \frac{50z^2 + 5z^3 - z^4}{6z + 144} \\
27. w = \frac{121z + 11z^2 - 2z^3}{13z + 338} & 28. w = \frac{72z^2 + 6z^3 - z^4}{7z + 196} \\
29. w = \frac{169z + 13z^2 - 2z^3}{169z + 13z^2 - 2z^3} & 30. w = \frac{98z^2 + 7z^3 - z^4}{98z^2 + 7z^3 - z^4}
\end{array}$$

10. Разложить в ряд Лорана в окрестности точки z_0 функцию.

$$\begin{array}{ll}
1. w = z \cos \frac{1}{z-2}, z_0 = 2. & 2. w = \sin \frac{z}{z-1}, z_0 = 1. \\
3. w = ze^{z/(z-5)}, z_0 = 5. & 4. w = \sin \frac{2z-7}{z+2}, z_0 = -2. \\
5. w = \cos \frac{z}{z-i}, z_0 = i. & 6. w = \sin \frac{5z}{z-2i}, z_0 = 2i. \\
7. w = \sin \frac{3z-i}{3z+i}, z_0 = -\frac{i}{3}. & 8. w = z \cos \frac{3z}{z-1}, z_0 = 1. \\
9. w = z \sin \frac{z}{z-1}, z_0 = 1. & 10. w = (z-3) \cos \pi \frac{z-3}{z}, z_0 = 0. \\
11. w = z^2 \sin \pi \frac{z+1}{z}, z_0 = 0. & 12. w = z \cos \frac{z}{z+2i}, z_0 = -2i. \\
13. w = \cos \frac{z^2-4z}{(z-2)^2}, z_0 = 2. & 14. w = \sin \frac{z+i}{z-i}, z_0 = i. \\
15. w = \sin \frac{z}{z-3}, z_0 = 3. & 16. w = ze^{1/(z-2)}, z_0 = 2. \\
17. w = e^{z/(z-3)}, z_0 = 3. & 18. w = \sin \frac{2z}{z-4}, z_0 = 4. \\
19. w = \sin \frac{z^2-4z}{(z-2)^2}, z_0 = 2. & 20. w = e^{(4z-2z^2)/(z-1)^2}, z_0 = 1. \\
21. w = ze^{\pi/(z-a)^2}, z_0 = a. & 22. w = ze^{\pi z/(z-\pi)}, z_0 = \pi. \\
23. w = z \sin \pi \frac{z+2}{z}, z_0 = 0. & 24. w = z \cos \pi \frac{z+3}{z-1}, z_0 = 1. \\
25. w = z \sin \frac{z+3}{z}, z_0 = 0. & 26. w = z \sin \frac{z^2-2z}{(z-1)^2}, z_0 = 1. \\
27. w = z \cos \frac{z}{z-3}, z_0 = 3. & 28. w = z \sin \pi \frac{z-1}{z-2}, z_0 = 2. \\
29. w = z \cos \frac{z}{z-5}, z_0 = 5. & 30. w = ze^{z/(z-4)}, z_0 = 4.
\end{array}$$

11. Определить тип особой точки $z = 0$ для функции.

$$1. w = \frac{e^{9z} - 1}{\sin z - z + \frac{z^3}{6}} \quad 2. w = z^3 e^{7/z^2} \quad 3. w = \frac{\sin 8z - 8z}{\cos z - 1 + \frac{z^2}{2}}$$

$$\begin{array}{lll}
4. w = \frac{\cos 7z - 1}{\operatorname{sh} z - z - \frac{z^3}{6}} & 5. w = \frac{\operatorname{sh} 6z - 6z}{\operatorname{ch} z - 1 - \frac{z^2}{2}} & 6. w = \frac{\operatorname{ch} 5z - 1}{e^z - 1 - z} \\
7. w = z \sin \frac{6}{z^2} & 8. w = \frac{e^{z^2} - 1}{\sin z - z + \frac{z^3}{6}} & 9. w = \frac{\sin z^2 - z^2}{\cos z - 1 + \frac{z^2}{2}} \\
10. w = \frac{\cos z^2 - 1}{\operatorname{sh} z - z - \frac{z^3}{6}} & 11. w = \frac{e^{5z} - 1}{\operatorname{ch} z - 1 - \frac{z^2}{2}} & 12. w = \frac{\sin 4z - 4z}{e^z - 1 - z} \\
13. w = z^4 \cos \frac{5}{z^2} & 14. w = \frac{\cos 3z - 1}{\sin z - z + \frac{z^3}{6}} & 15. w = \frac{\operatorname{sh} 2z - 2z}{\cos z - 1 + \frac{z^2}{2}} \\
16. w = \frac{\operatorname{ch} 2z - 1}{\operatorname{sh} z - z - \frac{z^3}{6}} & 17. w = \frac{e^{z^3} - 1}{\operatorname{ch} z - 1 - \frac{z^2}{2}} & 18. w = \frac{\sin z^3 - z^3}{e^z - 1 - z} \\
19. w = ze^{4/z^3} & 20. w = \frac{\cos z^3 - 1}{\sin z - z + \frac{z^3}{6}} & 21. w = \frac{e^{7z} - 1}{\cos z - 1 + \frac{z^2}{2}} \\
22. w = \frac{\sin 6z - 6z}{\operatorname{sh} z - z - \frac{z^3}{6}} & 23. w = z \sin \frac{3}{z^3} & 24. w = \frac{\cos 5z - 1}{\operatorname{ch} z - 1 - \frac{z^2}{2}} \\
25. w = \frac{\operatorname{sh} 4z - 4z}{e^z - 1 - z} & 26. w = \frac{\operatorname{ch} 3z - 1}{\sin z - z + \frac{z^3}{6}} & 27. w = \frac{e^{z^4} - 1}{\cos z - 1 + \frac{z^2}{2}} \\
28. w = \frac{\sin z^4 - z^4}{\operatorname{sh} z - z - \frac{z^3}{6}} & 29. w = z \cos \frac{2}{z^3} & 30. w = \frac{\cos z^4 - 1}{\operatorname{ch} z - 1 - \frac{z^2}{2}}
\end{array}$$

12. Найти вычеты во всех конечных особых точках функции.

$$\begin{array}{lll}
1. w = \frac{1}{z + z^3} & 2. w = \frac{z^2}{1 + z^4} & 3. w = \frac{z^2}{(z + 1)^3} \\
4. w = \frac{1}{(1 + z^2)^3} & 5. w = \frac{1}{(1 + z^2)(z - 1)^2} & 6. w = \frac{\cos z}{(z - 1)^2} \\
7. w = \frac{1}{e^z + 1} & 8. w = \frac{\sin \pi z}{(z - 1)^3} & 9. w = \frac{1}{z^6(z - 2)} \\
10. w = \frac{1 + z^8}{z^6(z + 2)} & 11. w = \frac{1 + z^{10}}{z^6(z^2 + 4)} & 12. w = \sin z \sin \frac{1}{z} \\
13. w = \frac{\cos z}{(z^2 + 1)^2} & 14. w = \frac{\sin z}{(z^2 + 1)^2} & 15. w = \frac{\cos 2z}{(z - 1)^3} \\
16. w = e^{3/(z-2)} & 17. w = \frac{z^2 + 1}{z - 2} & 18. w = \frac{z^2}{(z^2 + 1)^2} \\
19. w = \frac{\sin 2z}{(z + 1)^4} & 20. w = \frac{e^z}{z^2(z^2 + 9)} & 21. w = \frac{\cos^3 z}{z^3}
\end{array}$$

$$\begin{aligned}
22. w &= \frac{z^2 + z - 1}{z^2(z-1)}. & 23. w &= \frac{1}{z(1-z^2)}. & 24. w &= \frac{1}{z^2 - z^5}. \\
25. w &= \frac{\cos 4z}{(z-2)^6}. & 26. w &= \frac{\sin z^2}{z^2(z - \frac{\pi}{4})}. & 27. w &= z^2 e^{1/z}. \\
28. w &= \frac{e^z}{(z+1)^3(z-2)}. & 29. w &= (z-2)e^{1/(z-2)}. & 30. w &= \frac{z^2}{(z-2)^3}.
\end{aligned}$$

13. При помощи вычетов вычислить интеграл.

$$\begin{aligned}
1. \oint_{|z|=1/2} \frac{dz}{z(z^2+1)}. & \quad 2. \oint_{z-i=3/2} \frac{dz}{z(z^2+4)}. \\
3. \oint_{|z-1-i|=5/4} \frac{2dz}{z^2(z-1)}. & \quad 4. \oint_{|z|=1} \frac{2+\sin z}{z(z+2i)} dz. \\
5. \oint_{|z-3|=1/2} \frac{e^z dz}{\sin z}. & \quad 6. \oint_{z-3/2=i} \frac{z(\sin z+2)}{\sin z} dz. \\
7. \oint_{|z-1|=3} \frac{ze^z}{\sin z} dz. & \quad 8. \oint_{z+3/2=i} \frac{2z(z-1)}{\sin z} dz. \\
9. \oint_{|z-1/4|=1/3} \frac{z(z+1)^2}{\sin 2\pi z} dz. & \quad 10. \oint_{|z-1/2|=1} \frac{iz(z-i)}{\sin \pi z} dz. \\
11. \oint_{|z-3|=1} \frac{\sin 3z+2}{z^2(z-\pi)} dz. & \quad 12. \oint_{|z-1/2|=1} \frac{e^z+1}{z(z-1)} dz. \\
13. \oint_{|z|=1} \frac{(e^{z^i}+2)z}{\sin 3zi} dz. & \quad 14. \oint_{|z-2|=3} \frac{\cos^2 z+1}{z^2-\pi^2} dz. \\
15. \oint_{|z-1|=3/2} \frac{\ln(z+2)}{\sin z} dz. & \quad 16. \oint_{|z-6|=1} \frac{\sin^3 z+2}{z-4\pi^2} dz. \\
17. \oint_{|z+1|=2} \frac{\sin^2 z-3}{z^2+2\pi z} dz. & \quad 18. \oint_{|z+3/2|=1} \frac{\cos^2 z+3}{2z^2+\pi z} dz. \\
19. \oint_{|z+1|=1/2} \frac{\operatorname{tg} z+2}{4z^2+\pi z} dz. & \quad 20. \oint_{|z|=1/4} \frac{\ln(e+z)}{z \sin(z+\pi/4)} dz. \\
21. \oint_{|z=\pi/2} \frac{z^2+z+3}{(\sin z)(\pi+z)} dz. & \quad 22. \oint_{|z|=1} \frac{z^3-i}{(\sin 2z)(z-\pi)} dz. \\
23. \oint_{|z-1|=2} \frac{z(z+\pi)}{\sin 2z} dz. & \quad 24. \oint_{|z|=2} \frac{z^2+\sin z+2}{z^2+\pi z} dz. \\
25. \oint_{|z-3/2|=1} \frac{z(z+\pi)}{(\sin 3z)(z-\pi)} dz. & \quad 26. \oint_{|z-3/2|=2} \frac{\sin z dz}{z(z-\pi)(z+\pi/3)}.
\end{aligned}$$

$$27. \oint_{|z-\pi|=1} \frac{z^2 + \pi^2}{i \sin z} dz. \quad 28. \oint_{|z|=2} \frac{\sin^2 z}{z \cos z} dz.$$

$$29. \oint_{|z-\pi|=2} \frac{\cos^2 z}{z \sin z} dz. \quad 30. \oint_{|z-3/2|=2} \frac{z^3 + \sin 2z}{(\sin z/2)(z-\pi)} dz.$$

14. При помощи вычетов вычислить интеграл.

$$1. \int_{-\infty}^{+\infty} \frac{dx}{(x^2+1)^3}. \quad 2. \int_{-\infty}^{+\infty} \frac{x \sin x}{x^2-2x+10} dx. \quad 3. \int_{-\infty}^{+\infty} \frac{x \cos x}{x^2-2x+10} dx.$$

$$4. \int_{-\infty}^{+\infty} \frac{x^2+1}{x^4+1} dx. \quad 5. \int_{-\infty}^{+\infty} \frac{x \sin x}{x^2+4x+20} dx. \quad 6. \int_{-\infty}^{+\infty} \frac{x \cos x}{x^2+x+1} dx.$$

$$7. \int_0^{+\infty} \frac{\cos x}{x^2+9} dx. \quad 8. \int_0^{+\infty} \frac{\cos x}{x^4+5x^2+4} dx. \quad 9. \int_{-\infty}^{+\infty} \frac{(x+1) \sin 2x}{x^2+2x+2} dx.$$

$$10. \int_0^{+\infty} \frac{\cos 2x}{(x^2+1)^2} dx. \quad 11. \int_{-\infty}^{+\infty} \frac{(x-1) \cos 2x}{x^2-4x+5} dx. \quad 12. \int_{-\infty}^{+\infty} \frac{x \sin x}{x^2+2x+10} dx.$$

$$13. \int_0^{+\infty} \frac{x \sin x}{x^2+1} dx. \quad 14. \int_{-\infty}^{+\infty} \frac{(2x^3+13x) \sin x}{x^4+13x^2+36} dx. \quad 15. \int_{-\infty}^{+\infty} \frac{x^2}{(x^2+1)(x^2+9)} dx.$$

$$16. \int_{-\infty}^{+\infty} \frac{x^2+1}{x^4+1} dx. \quad 17. \int_{-\infty}^{+\infty} \frac{x^2-x+2}{x^4+10x^2+9} dx. \quad 18. \int_0^{+\infty} \frac{x^4}{(3x^2+2)^4} dx.$$

$$19. \int_{-\infty}^{+\infty} \frac{dx}{(x^2+1)^3}. \quad 20. \int_0^{+\infty} \frac{dx}{(x^2+4)(x^2+1)}. \quad 21. \int_0^{+\infty} \frac{x \sin 3x}{x^2+4} dx.$$

$$22. \int_{-\infty}^{+\infty} \frac{x^4+1}{x^6+1} dx. \quad 23. \int_{-\infty}^{+\infty} \frac{x^2}{(x^2+1)(x^2+9)} dx. \quad 24. \int_0^{+\infty} \frac{\cos x}{(x^2+1)(x^2+4)} dx.$$

$$25. \int_0^{+\infty} \frac{\cos x}{(x^2+9)^3} dx. \quad 26. \int_{-\infty}^{+\infty} \frac{(x^3+5x) \sin x}{x^4+10x^2+9} dx. \quad 27. \int_0^{+\infty} \frac{\cos 3x}{(x^2+1)^2} dx.$$

$$28. \int_0^{+\infty} \frac{x \sin 4x}{x^2+9} dx. \quad 29. \int_{-\infty}^{+\infty} \frac{dx}{(x^2+4)(x^2+1)^2}. \quad 30. \int_0^{+\infty} \frac{dx}{(x^2+9)(x^2+16)}.$$

15. При помощи вычетов вычислить интеграл.

$$1. \int_0^{2\pi} \frac{dt}{(2+\sqrt{3} \cos t)^2}. \quad 2. \int_0^{2\pi} \frac{dt}{(4+\sqrt{15} \cos t)^2}. \quad 3. \int_0^{2\pi} \frac{dt}{(5+2\sqrt{6} \cos t)^2}.$$

$$4. \int_0^{2\pi} \frac{dt}{(6+\sqrt{35} \cos t)^2}. \quad 5. \int_0^{2\pi} \frac{dt}{(7+4\sqrt{3} \cos t)^2}. \quad 6. \int_0^{2\pi} \frac{dt}{(5-4 \cos t)^2}.$$

$$\begin{array}{lll}
7. \int_0^{2\pi} \frac{dt}{(5-3\cos t)^2} & 8. \int_0^{2\pi} \frac{dt}{(8-3\sqrt{7}\cos t)^2} & 9. \int_0^{2\pi} \frac{dt}{(9-4\sqrt{5}\cos t)^2} \\
10. \int_0^{2\pi} \frac{dt}{(4-\sqrt{7}\cos t)^2} & 11. \int_0^{2\pi} \frac{dt}{(3-\sqrt{5}\cos t)^2} & 12. \int_0^{2\pi} \frac{dt}{(3-2\sqrt{2}\cos t)^2} \\
13. \int_0^{2\pi} \frac{dt}{(4-2\sqrt{3}\cos t)^2} & 14. \int_0^{2\pi} \frac{dt}{(5-\sqrt{21}\cos t)^2} & 15. \int_0^{2\pi} \frac{dt}{(6-4\sqrt{2}\cos t)^2} \\
16. \int_0^{2\pi} \frac{dt}{(8-2\sqrt{15}\cos t)^2} & 17. \int_0^{2\pi} \frac{dt}{(\sqrt{5}\cos t-2)^2} & 18. \int_0^{2\pi} \frac{dt}{(\sqrt{15}\cos t-4)^2} \\
19. \int_0^{2\pi} \frac{dt}{(2\sqrt{6}\cos t-5)^2} & 20. \int_0^{2\pi} \frac{dt}{(\sqrt{35}\cos t-6)^2} & 21. \int_0^{2\pi} \frac{dt}{(4\sqrt{3}\cos t-7)^2} \\
22. \int_0^{2\pi} \frac{dt}{(4\cos t+5)^2} & 23. \int_0^{2\pi} \frac{dt}{(3\cos t+5)^2} & 24. \int_0^{2\pi} \frac{dt}{(3\sqrt{7}\cos t+8)^2} \\
25. \int_0^{2\pi} \frac{dt}{(4\sqrt{5}\cos t+9)^2} & 26. \int_0^{2\pi} \frac{dt}{(\sqrt{7}\cos t+4)^2} & 27. \int_0^{2\pi} \frac{dt}{(\sqrt{5}\cos t+3)^2} \\
28. \int_0^{2\pi} \frac{dt}{(2\sqrt{2}\cos t+3)^2} & 29. \int_0^{2\pi} \frac{dt}{(2\sqrt{3}\cos t+4)^2} & 30. \int_0^{2\pi} \frac{dt}{(\sqrt{21}\cos t+5)^2}
\end{array}$$

16. Найти образ $F(p)$ по оригиналу $f(t) = e^{at} \sin^2 bt$.

- | | | |
|-----------------------|-----------------------|-----------------------|
| 1. $a = 1, b = 1.$ | 2. $a = 1, b = -1.$ | 3. $a = 1, b = 2.$ |
| 4. $a = 1, b = -2.$ | 5. $a = 1, b = 3.$ | 6. $a = 1, b = -3.$ |
| 7. $a = -1, b = 1.$ | 8. $a = -1, b = -1.$ | 9. $a = -1, b = 2.$ |
| 10. $a = -1, b = -3.$ | 11. $a = 1, b = 3.$ | 12. $a = -1, b = -3.$ |
| 13. $a = 2, b = 1.$ | 14. $a = 2, b = -1.$ | 15. $a = 2, b = 2.$ |
| 16. $a = 2, b = -2.$ | 17. $a = 2, b = 3.$ | 18. $a = 2, b = -3.$ |
| 19. $a = -2, b = 1.$ | 20. $a = -2, b = -1.$ | 21. $a = -2, b = 2.$ |
| 22. $a = -2, b = -2.$ | 23. $a = -2, b = 3.$ | 24. $a = -2, b = -3.$ |
| 25. $a = 3, b = 1.$ | 26. $a = 3, b = -1.$ | 27. $a = 3, b = 2.$ |
| 28. $a = 3, b = -2.$ | 29. $a = 3, b = 3.$ | 30. $a = 3, b = -3.$ |

17. Найти оригинал $f(t)$ по образу $F(p) = \frac{p-1}{(p-a)(p-b)}$.

- | | | |
|-----------------------|----------------------|-----------------------|
| 1. $a = 1, b = 1.$ | 2. $a = 1, b = -1.$ | 3. $a = 1, b = 2.$ |
| 4. $a = 1, b = -2.$ | 5. $a = 1, b = 3.$ | 6. $a = 1, b = -3.$ |
| 7. $a = -1, b = 1.$ | 8. $a = -1, b = -1.$ | 9. $a = -1, b = 2.$ |
| 10. $a = -1, b = -3.$ | 11. $a = 1, b = 3.$ | 12. $a = -1, b = -3.$ |
| 13. $a = 2, b = 1.$ | 14. $a = 2, b = -1.$ | 15. $a = 2, b = 2.$ |

16. $a = 2, b = -2.$ 17. $a = 2, b = 3.$ 18. $a = 2, b = -3.$
 19. $a = -2, b = 1.$ 20. $a = -2, b = -1.$ 21. $a = -2, b = 2.$
 22. $a = -2, b = -2.$ 23. $a = -2, b = 3.$ 24. $a = -2, b = -3.$
 25. $a = 3, b = 1.$ 26. $a = 3, b = -1.$ 27. $a = 3, b = 2.$
 28. $a = 3, b = -2.$ 29. $a = 3, b = 3.$ 30. $a = 3, b = -3.$

18. Найти оригинал $f(t)$ по образу $F(p) = \frac{ap+1}{p^2+bp+1}.$

1. $a = 1, b = 1.$ 2. $a = 1, b = -1.$ 3. $a = 1, b = 2.$
 4. $a = 1, b = -2.$ 5. $a = 1, b = 3.$ 6. $a = 1, b = -3.$
 7. $a = -1, b = 1.$ 8. $a = -1, b = -1.$ 9. $a = -1, b = 2.$
 10. $a = -1, b = -3.$ 11. $a = 1, b = 3.$ 12. $a = -1, b = -3.$
 13. $a = 2, b = 1.$ 14. $a = 2, b = -1.$ 15. $a = 2, b = 2.$
 16. $a = 2, b = -2.$ 17. $a = 2, b = 3.$ 18. $a = 2, b = -3.$
 19. $a = -2, b = 1.$ 20. $a = -2, b = -1.$ 21. $a = -2, b = 2.$
 22. $a = -2, b = -2.$ 23. $a = -2, b = 3.$ 24. $a = -2, b = -3.$
 25. $a = 3, b = 1.$ 26. $a = 3, b = -1.$ 27. $a = 3, b = 2.$
 28. $a = 3, b = -2.$ 29. $a = 3, b = 3.$ 30. $a = 3, b = -3.$

19. При помощи преобразования Лапласа решить задачу Коши.

1. $\ddot{x} + 3\dot{x} = e^t, x(0) = 0, \dot{x}(0) = -1.$
 2. $\ddot{x} + 2\dot{x} - 3x = e^{-t}, x(0) = 0, \dot{x}(0) = 1.$
 3. $\ddot{x} + 2\dot{x} + x = \sin t, x(0) = 0, \dot{x}(0) = -1.$
 4. $\ddot{x} - 2\dot{x} + x = e^t, x(0) = 0, \dot{x}(0) = 1.$
 5. $\ddot{x} - 2\dot{x} + 2x = 1, x(0) = 0, \dot{x}(0) = 0.$
 6. $\ddot{x} + \dot{x} = \cos t, x(0) = 2, \dot{x}(0) = 0.$
 7. $\ddot{x} + 2\dot{x} + x = t^2, x(0) = 1, \dot{x}(0) = 0.$
 8. $\ddot{x} + x = \cos t, x(0) = -1, \dot{x}(0) = 1.$
 9. $\ddot{x} + 2\dot{x} + 5x = 3, x(0) = 1, \dot{x}(0) = 0.$
 10. $\ddot{x} + 2\dot{x} + 2x = 1, x(0) = 0, \dot{x}(0) = 0.$
 11. $\ddot{x} + x = 1, x(0) = -1, \dot{x}(0) = 0.$
 12. $\ddot{x} + 4x = t, x(0) = 1, \dot{x}(0) = 0.$
 13. $\ddot{x} - 2\dot{x} + 5x = 1 - t, x(0) = 0, \dot{x}(0) = 0.$
 14. $\ddot{x} + \dot{x} = \cos t, x(0) = 2, \dot{x}(0) = 0.$
 15. $\ddot{x} - \dot{x} = te^t, x(0) = 0, \dot{x}(0) = 0.$
 16. $\ddot{x} + 2\dot{x} + x = t, x(0) = 0, \dot{x}(0) = 0.$
 17. $\ddot{x} - \dot{x} = t^2, x(0) = 0, \dot{x}(0) = 0.$
 18. $\ddot{x} - x = \sin t, x(0) = -1, \dot{x}(0) = 0.$
 19. $\ddot{x} + x = 2 \sin t, x(0) = 1, \dot{x}(0) = -1.$
 20. $\ddot{x} - 2\dot{x} + x = t - \sin t, x(0) = 0, \dot{x}(0) = 0.$
 21. $\ddot{x} + 2\dot{x} + x = 2 \cos^2 t, x(0) = 0, \dot{x}(0) = 0.$
 22. $\ddot{x} + x = te^t + 4 \sin t, x(0) = 0, \dot{x}(0) = 0.$

23. $\ddot{x} - 3\dot{x} + 2x = e^t, x(0) = 0, \dot{x}(0) = 0.$
 24. $\ddot{x} - \dot{x} = t^2, x(0) = 0, \dot{x}(0) = 1.$
 25. $\ddot{x} + x = t \cos 2t, x(0) = 0, \dot{x}(0) = 0.$
 26. $\ddot{x} + 4x = \sin t, x(0) = 0, \dot{x}(0) = 0.$
 27. $\ddot{x} + \dot{x} = t \cos t, x(0) = 0, \dot{x}(0) = 0.$
 28. $\ddot{x} - 3\dot{x} + 2x = e^{5t}, x(0) = 1, \dot{x}(0) = 2.$
 29. $\ddot{x} - 2\dot{x} = e^{2t}, x(0) = 0, \dot{x}(0) = 0.$
 30. $\ddot{x} + x = 2 \cos t, x(0) = 0, \dot{x}(0) = -1.$

20. При помощи преобразования Лапласа решить систему.

1.
$$\begin{cases} \dot{x} = x + 3y + 2 \\ \dot{y} = x - y + 1, x(0) = -1, y(0) = 2 \end{cases}$$
2.
$$\begin{cases} \dot{x} = -x + 3y + 1 \\ \dot{y} = x + y, x(0) = 1, y(0) = 2 \end{cases}$$
3.
$$\begin{cases} \dot{x} = x + 4y \\ \dot{y} = 2x - y + 9, x(0) = 1, y(0) = 0 \end{cases}$$
4.
$$\begin{cases} \dot{x} = x + 2y + 1 \\ \dot{y} = 4x - y, x(0) = 0, y(0) = 1 \end{cases}$$
5.
$$\begin{cases} \dot{x} = 2x + 5y \\ \dot{y} = x - 2y + 2, x(0) = 1, y(0) = 1 \end{cases}$$
6.
$$\begin{cases} \dot{x} = -2x + 5y + 1 \\ \dot{y} = x + 2y + 1, x(0) = 0, y(0) = 2 \end{cases}$$
7.
$$\begin{cases} \dot{x} = 3x + y \\ \dot{y} = -5x - 3y + 2, x(0) = 2, y(0) = 0 \end{cases}$$
8.
$$\begin{cases} \dot{x} = -3x - 4y + 1 \\ \dot{y} = 2x + 3y, x(0) = 0, y(0) = 2 \end{cases}$$
9.
$$\begin{cases} \dot{x} = -2x + 6y + 1 \\ \dot{y} = 2x + 2y, x(0) = 0, y(0) = 1 \end{cases}$$
10.
$$\begin{cases} \dot{x} = 2x + 3y + 1 \\ \dot{y} = 4x - 2y, x(0) = -1, y(0) = 0 \end{cases}$$
11.
$$\begin{cases} \dot{x} = x + 2y \\ \dot{y} = 2x + y + 1, x(0) = 0, y(0) = 5 \end{cases}$$
12.
$$\begin{cases} \dot{x} = 2x - 2y \\ \dot{y} = -4x, x(0) = 3, y(0) = 1 \end{cases}$$
13.
$$\begin{cases} \dot{x} = -x - 2y + 1 \\ \dot{y} = -\frac{3}{2}x + y, x(0) = 1, y(0) = 0 \end{cases}$$
14.
$$\begin{cases} \dot{x} = 3x + 5y + 2 \\ \dot{y} = 3x + y + 1, x(0) = 0, y(0) = 2 \end{cases}$$
15.
$$\begin{cases} \dot{x} = 3x + 2y \\ \dot{y} = \frac{5}{2}x - y + 2, x(0) = 0, y(0) = 1 \end{cases}$$

16. $\begin{cases} \dot{x} = 2y + 1 \\ \dot{y} = 2x + 3, x(0) = -1, y(0) = 0 \end{cases}$
17. $\begin{cases} \dot{x} = 2x + 8y + 1 \\ \dot{y} = 3x + 4y, x(0) = 2, y(0) = 1 \end{cases}$
18. $\begin{cases} \dot{x} = 2x + 2y + 2 \\ \dot{y} = 4y + 1, x(0) = 0, y(0) = 1 \end{cases}$
19. $\begin{cases} \dot{x} = x + y \\ \dot{y} = 4x + y + 1, x(0) = 1, y(0) = 0 \end{cases}$
20. $\begin{cases} \dot{x} = x - 2y + 1 \\ \dot{y} = -3x, x(0) = 0, y(0) = 1 \end{cases}$
21. $\begin{cases} \dot{x} = 3y + 2 \\ \dot{y} = x + 2y, x(0) = -1, y(0) = 1 \end{cases}$
22. $\begin{cases} \dot{x} = x + 4y + 1 \\ \dot{y} = 2x + 3y, x(0) = 0, y(0) = 1 \end{cases}$
23. $\begin{cases} \dot{x} = 2y \\ \dot{y} = 2x + 3y + 1, x(0) = 2, y(0) = 1 \end{cases}$
24. $\begin{cases} \dot{x} = -2x + y + 2 \\ \dot{y} = 3x, x(0) = 1, y(0) = 0 \end{cases}$
25. $\begin{cases} \dot{x} = 4x + 3 \\ \dot{y} = x + 2y, x(0) = -1, y(0) = 0 \end{cases}$
26. $\begin{cases} \dot{x} = y + 3 \\ \dot{y} = x + 2, x(0) = 1, y(0) = 0 \end{cases}$
27. $\begin{cases} \dot{x} = x + 3y + 3 \\ \dot{y} = x - y + 1, x(0) = 0, y(0) = 1 \end{cases}$
28. $\begin{cases} \dot{x} = -x + 3y + 2 \\ \dot{y} = x + y + 1, x(0) = 0, y(0) = 1 \end{cases}$
29. $\begin{cases} \dot{x} = 3y \\ \dot{y} = 3x + 1, x(0) = 2, y(0) = 0 \end{cases}$
30. $\begin{cases} \dot{x} = x + 3y \\ \dot{y} = x - y, x(0) = 1, y(0) = 0 \end{cases}$